NAG C Library Function Document

nag_dpbstf (f08ufc)

1 Purpose

nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive-definite band matrix.

2 Specification

```
void nag_dpbstf (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer kb,
     double bb[], Integer pdbb, NagError *fail)
```

3 Description

nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive-definite band matrix B. It is designed to be used in conjunction with nag_dsbgst (f08uec).

The factorization has the form $B = S^T S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n+k)/2$ rows, and transposed hence, lower triangular in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & s_{64} & s_{65} & s_{66} & & & \\ & & & & s_{75} & s_{76} & s_{77} & & \\ & & & & & s_{86} & s_{87} & s_{88} & \\ & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

4 References

None.

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether the upper or lower triangular part of B is stored as follows:

if **uplo** = **Nag_Upper**, the upper triangular part of B is stored;

if **uplo** = **Nag_Lower**, the lower triangular part of B is stored.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix B.

Constraint: **n** ≥ 0.

4: **kb** – Integer *Input*

On entry: k, the number of super-diagonals of the matrix B if **uplo** = **Nag_Upper**, or the number of sub-diagonals if **uplo** = **Nag_Lower**.

Constraint: **kb** ≥ 0.

5: **bb**[dim] – double *Input/Output*

Note: the dimension, dim, of the array **bb** must be at least $\max(1, \textbf{pdbb} \times \textbf{n})$.

On entry: the n by n symmetric band matrix B. This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements b_{ij} depends on the **order** and **uplo** parameters as follows:

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Upper**,
b_{ij} is stored in **bb**[$k + i - j + (j-1) \times$ **pdbb**], for $i = 1, \ldots, n$ and $j = i, \ldots, \min(n, i+k)$;

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Lower**,
b_{ij} is stored in **bb**[$i - j + (j-1) \times$ **pdbb**], for $i = 1, \ldots, n$ and $j = \max(1, i-k), \ldots, i$;

if **order** = **Nag_RowMajor** and **uplo** = **Nag_Upper**,
b_{ij} is stored in **bb**[$j - i + (i-1) \times$ **pdbb**], for $i = 1, \ldots, n$ and $j = i, \ldots, \min(n, i+k)$;

if **order** = **Nag_RowMajor** and **uplo** = **Nag_Lower**,
b_{ij} is stored in **bb**[$k + j - i + (i-1) \times$ **pdbb**], for $i = 1, \ldots, n$ and $j = \max(1, i-k), \ldots, i$.

On exit: B is overwritten by the elements of its split Cholesky factor S.

6: **pdbb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix B in the array **bb**.

Constraint: **pdbb** $\geq$ **kb** $+ 1$.

7: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **kb** = ⟨*value*⟩.
Constraint: **kb** ≥ 0.

On entry, **pdbb** = ⟨*value*⟩.
Constraint: **pdbb** > 0.

NE_INT_2

On entry, **pdbb** = ⟨*value*⟩, **kb** = ⟨*value*⟩.
Constraint: **pdbb** $\geq$ **kb** $+ 1$.

NE_CONVERGENCE

The factorization could not be completed, because updated element $b(\langle value \rangle, \langle value \rangle)$ would be the square root of a negative number. Hence B is not positive definite. This may indicate an error in forming the matrix B.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix $B + E$, where

$$|E| \leq c(k+1)\varepsilon |S^T|\,|S|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ε is the ***machine precision***. It follows that $|e_{ij}| \leq c(k+1)\varepsilon\sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $n(k+1)^2$, assuming $n \gg k$.

A call to this function may be followed by a call to nag_dsbgst (f08uec) to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive-definite.

The complex analogue of this function is nag_zpbstf (f08utc).

9 Example

See Section 9 of the document for nag_dsbgst (f08uec).